# The Higgs Mechanism and The Vacuum Energy Density Problem


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We discuss the vacuum energy density term resulting from the spontaneous breakdown of the electroweak gauge symmetry, in the Higgs Mechanism. We alternatively expand the scalar field at one of the degenerate states that lie outside the circle of minimum, such that the Higgs Potential becomes free of any constant field term, and describes a true vacuum. We show that this true vacuum requires a slightly smaller quartic coupling, if the same $v$ and $m_H$ values of the electroweak model are imposed. We propose that this small difference ( exactly 20 percent ) can be utilized as a test to distinguish and identify the true vacuum, in future experiments at LHC. We shortly discuss the resulting new Higgs Potential, and its cosmological implications.



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## I. INTRODUCTION

The Higgs Mechanism [1][2] implemented in the Electroweak Model [3] [4] [5] is so far the only tool that can generate masses for gauge bosons and fermions [6]. However it predicts the existence of a massive Higgs boson that still awaits experimental confirmation. Yet another pending problem in the Higgs Mechanism is that it generates a constant term which contributes to the energy density of the vacuum. The magnitude of the contribution can be estimated. Thus a Higgs mass slightly above the current lower bound, leads to an energy density roughly 55 orders of magnitude larger than the current value. It is very hard to account for such a tremendous mismatch between theory and observation. In this paper we argue that this contribution might follow from our miss understanding of the mechanism. A brief review of the Higgs Mechanism is therefore essential for our discussion. We start with the lagrangian of the spinless scalar field

$$L_\phi = (D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\left(\phi^\dagger\phi\right) - \lambda\left(\phi^\dagger\phi\right)^2 \tag{1}$$

where $\phi$ is a doublet of complex scalar fields transforming under the $SU(2)\times U(1)_Y$ electroweak gauge symmetry and $D_\mu$ is the gauge covariant derivative.

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \tag{2}$$

The electric charges of the components in $\phi$ follow from $Q = I_3 + Y/2$, and the components are made up of 4 real valued scalar fields

$$\phi^+ = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad , \quad \phi^0 = \frac{\phi_3 + i\phi_4}{\sqrt{2}} \tag{3}$$

The potential part of the Higgs lagrangian has a minimum when $\mu^2 > 0$ and $\lambda > 0$.

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \tag{4}$$

The minima follows from the functional derivative $\partial V/\partial\phi = 0$. This gives us the condition : $\phi^\dagger\phi = \mu^2/2\lambda$. A particular solution of the minimum denoted with $\phi_0$ is

$$\phi_1 = \phi_2 = \phi_4 = 0,\ \phi_3^2 = \frac{\mu^2}{\lambda} = v^2,\ \phi_0 = \begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix} \tag{5}$$

The scalar field $\phi$ could be expanded around the minimum $\phi_0$ with new field variables $\xi$ and $H$, the former fields give rise to massless 'would-be' goldstone bosons [7] [8] and the latter is the massive Higgs field. A useful expression for the scalar field expanded around the minimum that eliminates the $\xi$ fields is

$$\phi(x) \approx \phi_0(x) = e^{-i\,\sigma\cdot\xi(x)/v}\begin{pmatrix}0\\ \frac{v+H(x)}{\sqrt{2}}\end{pmatrix} \tag{6}$$

The complex scalar field is thereby reduced solely to the massive Higgs field $H$ by means of the unitary gauge [6]. If the above expression for $\phi(x)$ is substituted back in $V(\phi)$ we find the Higgs potential around the minimum

$$V(H) = \frac{1}{2}m_H^2H^2 \pm \sqrt{\lambda}\mu H^3 + \frac{\lambda}{4}H^4 - \frac{\mu^4}{4\lambda} \tag{7}$$

Herein the first term gives us the mass of the Higgs particle and the last term is a constant, contributing to the field. We have

$$m_H^2 = 2\mu^2 > 0,\ v = \pm\frac{\mu}{\sqrt{\lambda}},\ V(0) = -\frac{\mu^4}{4\lambda} = -\frac{1}{8}m_H^2v^2 \tag{8}$$

We have deliberately kept the $\pm$ sign in $v$. It affects only the term with $H^3$ in $V(H)$, other terms do not suffer any change in sign. Finally, $V(0)$ is equal to $V(H)$ at $H = 0$. The Higgs potential has the following feature at $H = 0$.

$$\begin{aligned}\left(\frac{\partial V}{\partial H}\right)_0 &= 0\\ \left(\frac{\partial^2 V}{\partial H^2}\right)_0 &= 2\mu^2 = m_H^2 > 0\end{aligned} \tag{9}$$


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## II. DISCUSSION ABOUT THE VACUUM

As we wanted to expand the scalar field around the minimum of the potential $V(\phi)$, we informidably assigned the scalar field the value $\phi = \phi_0$. The potential $V(\phi)$ at the minimum assumes the value

$$V(\phi_0) = -\frac{\mu^4}{4\lambda} = -\frac{1}{8} m_H^2 v^2 \tag{10}$$

and has dimensions of energy density. A rough estimate of its value gives

$$-\frac{1}{8} m_H^2 v^2 \approx -2 \cdot 10^8 \mathrm{GeV}^4 \tag{11}$$

Here $m_H$ is chosen to be approximately 150 GeV, and $v = 246$ GeV. There is so far *no* way to get rid of this term[14], therefore it should somehow manifest itself in our universe. Direct calculations from cosmology, give us a vacuum energy density that is roughly 55 orders smaller [9][10]. We could say that the vacuum energy density is practically zero. As a result the above term is obviously indicating that there is something wrong with either the Higgs Mechanism, or the way we make use of it[15]. Note that there is a negative sign appearing in the term, independent of the sign of $v$. Since energy should always be positive ,the term also fails to describe a well defined physical quantity. Therefore our interpretations is that the term basically indicates the shift between the two levels

$$V(\phi = 0) \leftrightarrow V(\phi = \phi_0) \tag{12}$$

where $V(\phi)$ is the scalar potential in eq. (4).

## III. MODIFYING THE HIGGS MECHANISM

We investigate the symmetry breaking procedure where the scalar field is expanded, not around its minimum but around a state which satisfies the following condition

$$V(\phi_0) = 0, \qquad \phi_0 \neq 0 \tag{13}$$

Actually the above condition tells us that the scalar field should gain some expectation value without *shifting* the Vacuum. The usual assignment of the vacuum expectation value in the Higgs mechanism, feels itself free in doing so, and does not respect the vacuum. Therefore we end up with a huge energy density *missing* in the vacuum. The possible states $\phi_0$ that satisfy the above condition, lie on a circle, but do not correspond to the minimum of the scalar potential. It will be shown that this choice is not arbitrary and has measurable effects, when quartic Higgs interactions are considered. A discussion is postponed to the end of this section after the results are obtained.

The expectation value that satisfies the above condition is then found from $\phi_0^\dagger \phi_0 = \mu^2/\lambda$. A particular solution that satisfies $\phi_0$ is

$$\phi_1 = \phi_2 = \phi_4 = 0,\ \phi_3^2 = \frac{2\mu^2}{\lambda} = v^2,\ \phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{14}$$

The scalar field can be expanded around this particular $\phi_0$ by parameterizing it with $\xi$ fields and the $H$ field. The $\xi$ fields can be removed in the unitary gauge as before through

$$\phi(x) \approx \phi_0(x) = e^{-i\,\sigma\cdot\xi(x)/v} \begin{pmatrix} 0 \\ \frac{v+H(x)}{\sqrt{2}} \end{pmatrix} \tag{15}$$

By substituting the Higgs field in the scalar potential we obtain

$$V(H) = \pm\frac{\sqrt{2}\mu^3}{\sqrt{\lambda}} H + \frac{5}{2}\mu^2 H^2 \pm \mu\sqrt{2\lambda} H^3 + \frac{\lambda}{4} H^4 \tag{16}$$

We found out that the expression for the Higgs mass is changed, the energy density term disappeared and we have a new term proportional to $H$, which signifies the production of the Higgs Boson and was not present in the former Higgs potential. We have then

$$m_H^2 = 5\,\mu^2 > 0, \quad v = \pm\frac{\sqrt{2}\mu}{\sqrt{\lambda}}, \quad V(0) = 0 \tag{17}$$

For the Higgs potential $V(H)$ , the following two conditions occur

$$\begin{aligned} \left(\frac{\partial V}{\partial H}\right)_0 &= \pm\frac{\sqrt{2}\mu^3}{\sqrt{\lambda}} = \frac{m_H^2\, v}{5} \neq 0 \\ \left(\frac{\partial^2 V}{\partial H^2}\right)_0 &= +5\mu^2 = m_H^2 > 0 \end{aligned} \tag{18}$$

In comparison to the usual Higgs potential in eq.(9), it is seen that the first term above is differently non zero whereas the second term is the same.

At this stage it would be relevant to consider perturbations around the vacuum described by eq.(16). If we consider the system as made of a one dimensional *classical* potential such that

$$\begin{aligned} &V = a\,x + b\,x^2 + c\,x^3 + d\,x^4 \quad , \\ &a = \frac{m_H^2 v}{5}, \quad b = \frac{m_H^2}{2}, \quad c = v\lambda, \quad d = \frac{\lambda}{4} \end{aligned} \tag{19}$$

then oscillations would take place on top of a constant amplitude[16] $x_0 = \frac{a}{2b} \approx O(v)$, for small $\lambda$. As a result the ground state of the modified Higgs potential in eq.(16) contains the information that the vacuum intrinsically has an expectation value at the order of $v$, and perturbations take place on top of it. In contrast, a similar classical approach for the usual higgs potential in eq.(7), which describes a classical potential with $a = 0$,

would give instead $x_0 = 0$, because it lacks of a first order term in $H$. Thereby it is seen that the ground state of the usual Higgs potential does not reflect the vacuum expectation value (vev) i.e., perturbations occur around a vacuum state that appears to be void of any vev.

From the other side a 1-loop renormalization of *both* Higgs potentials will lead to a relatively small shift in $v$, which arises from the contributions of the tadpole diagrams [12]. Except for the divergent $-\frac{\mu^4}{4\lambda}$ term in the usual Higgs potential, the renormalization of the modified Higgs potential will differ from the renormalization of the usual Higgs potential only due to the tadpole linear in $H$, and is harmless in the $\phi^4$ theory.

If we assume that $v$ and $m_H$ are both in the *modified* and in the *usual* Higgs mechanism the same, then the respective quartic couplings $\lambda_M$ and $\lambda_U$ will differ in magnitude from each other, where the subscripts $U$ and $M$ ( M stands for modified and U for usual ) denote the respective parameters in eq.(8) and eq.(17). The Higgs mass in the usual and modified potential turns out to be $m_H = \sqrt{2\lambda_U}v$ and $m_H = \sqrt{\frac{5}{2}\lambda_M}v$ respectively. The value of $v$ is fixed through the Fermi Constant. A comparison of the quartic couplings without knowing the Higgs mass $m_H$ is possible, eliminating $m_H$ thus yields

$$\lambda_M = \frac{4}{5}\lambda_U \tag{20}$$

As a result our condition in eq.(13) predicts a slightly smaller quartic coupling, for the same $v$ and $m_H$. If future experiments at LHC find out a deviation from $\lambda_U$ in the electroweak model, precisely matching the above deficiency in the ratio, then this might be attributed to the condition in eq.(13).

## IV. CONCLUSION

In the state of the art we got rid of the energy density term by introducing the above condition, and got confronted with a linear term in $H$, which explicitly shows the creation of a massive scalar boson, that was not present before.

The modification that we imposed on the Higgs mechanism might be understood as a dynamical constraint on the vacuum, which prohibits the emergence of a shift term, and allows classical oscillations occur around the true vacuum. By *true* we describe a vacuum that has spontaneously received an expectation value $v$. This true vacuum meets the predictions of current cosmology which tells us that "vacuum" is free of any sizable energy content and leads to a vanishing cosmological constant. We proposed a test by means of measuring the quartic coupling. This test can justify whether the true vacuum and the ground state in nature is really described as in the modified Higgs mechanism or not.

We know that the energy density term in the Higgs mechanism is divergent [13] and doesn't influence observables, but there is no principle that prohibits it from coupling to gravity [10] despite of its negative sign. The condition in eq.(13) naturally hinders this catastrophe.

Finally, how the vacuum evolved from the unstable configuration to its current state which we described by eq.(13) is still not well understood and studied in models of inflation [10].


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[14] It is frequently argued in literature that adding a constant term to the lagrangian, which in principle *could* remove the term, is *not* a remedy. We subscribe to this point of view, since it is not natural.
[15] Another possibility, is to consider the spontaneous breakdown of the symmetry within the dynamics of an expanding universe[11], where the driving agent is the vacuum energy. Even if this were the case it would be impossible to understand why the expansion rate is *currently* so low for such a huge constant term, because it should be noted here that Higgs potential in eq.(7) describes, at $H \approx 0$ the *current* vacuum energy of our universe, and not that in the *past*. The current cosmological constant is not in favor of this huge energy. A zero vacuum energy, that would confirm the low expansion rate, could be obtained at $H = \mu/\sqrt{\lambda}$, ( for $v = -\mu/\sqrt{\lambda}$ ), but this identically washes all mass terms in the standard model lagrangian out.
[16] Here $x_0$ is the constant amplitude resulting from the inhomogeneous part. We show here that $x_0$ is classically at the order of the vacuum expectation value $v$.